STEPENI

$$\left(\frac{3a^{-x}}{1-a^{-x}}-\frac{2a^{-x}}{1+a^{-x}}-\frac{a^{x}}{a^{2x}-1}\right):\frac{a^{-x}}{a^{x}-a^{-x}}=$$

$$\left(\frac{\frac{3}{a^{x}}}{1-\frac{1}{a^{x}}}-\frac{\frac{2}{a^{x}}}{1+\frac{1}{a^{x}}}-\frac{a^{x}}{a^{2x}-1}\right):\frac{\frac{1}{a^{x}}}{a^{x}-\frac{1}{a^{x}}}=$$

$$\left(\frac{\frac{3}{a^{x}}}{\frac{a^{x}-1}{a^{x}}}-\frac{\frac{2}{a^{x}}}{\frac{a^{x}+1}{a^{x}}}-\frac{a^{x}}{a^{2x}-1}\right):\frac{\frac{1}{a^{x}}}{\frac{a^{2x}-1}{a^{x}}}=$$

$$\left(\frac{3}{a^x-1}-\frac{2}{a^x+1}-\frac{a^x}{(a^x-1)(a^x+1)}\right):\frac{1}{a^{2x}-1}=$$

$$\frac{3(a^x+1)-2(a^x-1)-a^x}{(a^x-1)(a^x+1)}\cdot\frac{a^{2x}-1}{1}=$$

$$\frac{3a^x+3-2a^x+2-a^x}{\cancel{(a^x-1)(a^x+1)}}\cdot\frac{\cancel{(a^x-1)(a^x+1)}}{1}=$$

$$=3+2=5$$

$$\left(\frac{x-x^{-2}}{x^{-2}+x^{-1}+1}-\frac{x-x^{-1}}{1+x^{-2}+2\cdot x^{-1}}\right):\frac{1-x^{-1}}{1+x^{-1}}$$

$$\left(\frac{a^n}{1-a^{-n}}+\frac{a^{-n}}{1+a^{-n}}\right)-\left(\frac{a^n}{1+a^{-n}}+\frac{a^{-n}}{1-a^{-n}}\right)$$

KORIJENI

1) $\frac{1}{\sqrt{5}}=\frac{1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{5}}{\sqrt{5^2}}=\frac{\sqrt{5}}{5}$

2) $\frac{9}{\sqrt{12}}=\frac{9}{\sqrt{12}}\cdot\frac{\sqrt{12}}{\sqrt{12}}=\frac{9\sqrt{12}}{12}=\frac{3\sqrt{12}}{4}=\frac{3\sqrt{4\cdot 3}}{4}=\frac{3\cdot 2\sqrt{3}}{4}=\frac{3\sqrt{3}}{2}$

3) $\frac{15}{2\sqrt{3}}=\frac{15}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{15\sqrt{3}}{2\cdot 3}=\frac{5\sqrt{3}}{2}$

$\frac{6}{\sqrt[3]{2}}=\frac{6}{\sqrt[3]{2}}\cdot\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}=\frac{6\cdot\sqrt[3]{2^2}}{\sqrt[3]{2^3}}=\frac{6\cdot\sqrt[3]{4}}{2}=3\sqrt[3]{4}$

5) $\frac{10}{\sqrt[4]{3}}=\frac{10}{\sqrt[4]{3}}\cdot\frac{\sqrt[4]{3^3}}{\sqrt[4]{3^3}}=\frac{10\sqrt[4]{27}}{\sqrt[4]{3^4}}=\frac{10\sqrt[4]{27}}{3}$

6) $\frac{ab}{\sqrt[3]{a^2b}}=\frac{ab}{\sqrt[3]{a^2b^1}}\cdot\frac{\sqrt[3]{a^1b^2}}{\sqrt[3]{a^1b^2}}=\frac{ab\sqrt[3]{ab^2}}{\sqrt[3]{a^3b^3}}=\frac{ab\sqrt[3]{ab^2}}{ab}=\sqrt[3]{ab^2}$

$$(A-B)\cdot(A+B)=A^2-B^2$$

$$A^3-B^3=(A-B)(A^2+AB+B^2)\rightarrow \text{Razlika kubova}$$

$$A^3+B^3=(A+B)(A^2-AB+B^2)\rightarrow \text{Zbir kubova}$$

7) $\frac{1}{2+\sqrt{3}}=\frac{1}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{2-\sqrt{3}}{2^2-\sqrt{3}^2}=\frac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$

8) $\frac{11}{\sqrt{6}-\sqrt{2}}=\frac{11}{\sqrt{6}-\sqrt{2}}\cdot\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}=\frac{11(\sqrt{6}+\sqrt{2})}{\sqrt{6}^2-\sqrt{2}^2}=\frac{11(\sqrt{6}+\sqrt{2})}{6-2}=\frac{11(\sqrt{6}+\sqrt{2})}{4}$

9)

$$\frac{5}{2\sqrt{3}-3\sqrt{2}}=\frac{5}{2\sqrt{3}-3\sqrt{2}}\cdot\frac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}+3\sqrt{2}}=\frac{5(2\sqrt{3}+3\sqrt{2})}{(2\sqrt{3})^2-(3\sqrt{2})^2}=\frac{5(2\sqrt{3}+3\sqrt{2})}{4\cdot3-9\cdot2}=\frac{5(2\sqrt{3}+3\sqrt{2})}{12-18}=\frac{5(2\sqrt{3}+3\sqrt{2})}{-6}$$

10)

$$\frac{1}{\sqrt[3]{3}+\sqrt[3]{2}}=\frac{1}{\sqrt[3]{3}+\sqrt[3]{2}}\cdot\frac{\sqrt[3]{3^2}-\sqrt[3]{3}\sqrt[3]{2}+\sqrt[3]{2^2}}{\sqrt[3]{3^2}-\sqrt[3]{3}\sqrt[3]{2}+\sqrt[3]{2^2}}=\frac{\sqrt[3]{9}-\sqrt[3]{6}+\sqrt[3]{4}}{\sqrt[3]{3}^3+\sqrt[3]{2}^3}=\frac{\sqrt[3]{9}-\sqrt[3]{6}+\sqrt[3]{4}}{3+2}=\frac{\sqrt[3]{9}-\sqrt[3]{6}+\sqrt[3]{4}}{5}$$

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11)

$$\frac{5}{\sqrt[3]{5}-\sqrt[3]{4}}=\frac{5}{\sqrt[3]{5}-\sqrt[3]{4}}\cdot\frac{\sqrt[3]{5^2}+\sqrt[3]{5}\sqrt[3]{4}+\sqrt[3]{4^2}}{\sqrt[3]{5^2}+\sqrt[3]{5}\sqrt[3]{4}+\sqrt[3]{4^2}}=\frac{5(\sqrt[3]{25}+\sqrt[3]{20}+\sqrt[3]{16})}{\sqrt[3]{5}^3-\sqrt[3]{4}^3}=5(\sqrt[3]{25}+\sqrt[3]{20}+\sqrt[3]{16})$$

LAGRANŽOV INDENTITET:

$$\boxed{\sqrt{a\pm\sqrt{b}}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\frac{a-\sqrt{a^2-b}}{2}}}\qquad a>0,\quad b>0,\quad b<a^2$$

a) $\sqrt{2+\sqrt{3}}=\sqrt{\frac{2+\sqrt{2^2-3}}{2}}+\sqrt{\frac{2-\sqrt{2^2-3}}{2}}$

$=\sqrt{\frac{2+\sqrt{1}}{2}}+\sqrt{\frac{2-\sqrt{1}}{2}}$

$=\sqrt{\frac{3}{2}}+\sqrt{\frac{1}{2}}=\frac{\sqrt{3}+1}{\sqrt{2}}$

$\sqrt{6-4\sqrt{2}}=\sqrt{6-\sqrt{16\cdot2}}=\sqrt{6-\sqrt{32}}=\sqrt{\frac{6+\sqrt{6^2-32}}{2}}+\sqrt{\frac{6-\sqrt{6^2-32}}{2}}$

$=\sqrt{\frac{6+\sqrt{36-32}}{2}}+\sqrt{\frac{6-\sqrt{36-32}}{2}}$

$=\sqrt{\frac{6+2}{2}}+\sqrt{\frac{6-2}{2}}=\sqrt{4}-\sqrt{2}=2-\sqrt{2}$

Dokazati da je: $\frac{4+2\sqrt{3}}{\sqrt[3]{10+6\sqrt{3}}}=\sqrt{3}+1$.

Racionalisati: $\frac{6}{\sqrt{21}+\sqrt{7}+2\sqrt{3}+2}$

Racionalisati: $\frac{1}{\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}}$

$\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\frac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}$